Exercise 64

(a) If $f(x) = 4x - \tan x$, $-\pi/2 < x < \pi/2$, find f' and f''.

(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

Solution

Part (a)

Calculate the first derivative of $f(x)$.

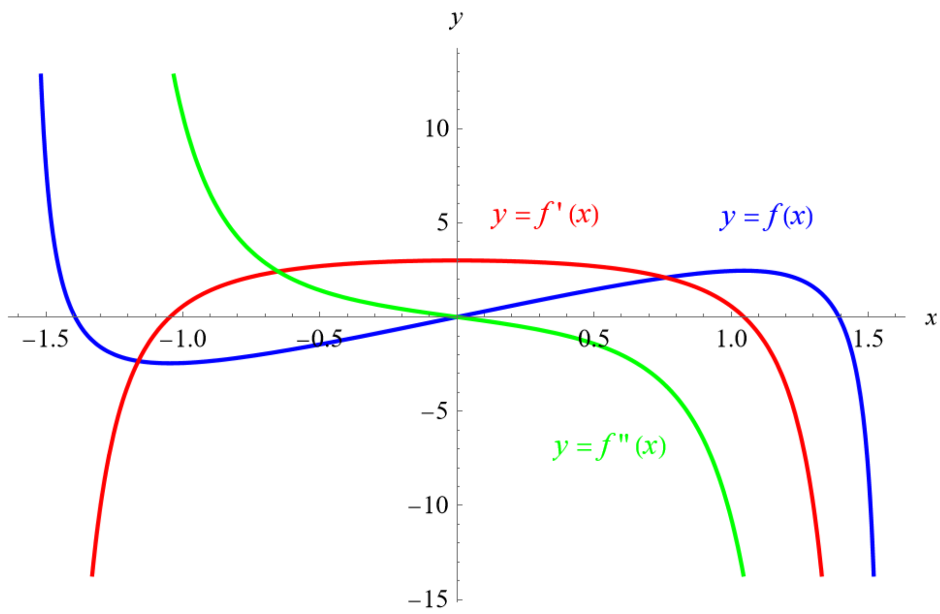
$$\begin{aligned} f'(x) &= \frac{d}{dx}(4x - \tan x) \\ &= \frac{d}{dx}(4x) - \frac{d}{dx}(\tan x) \\ &= (4) - (\sec^2 x) \\ &= 4 - \sec^2 x \end{aligned}$$

Calculate the second derivative of $f(x)$.

$$\begin{aligned} f''(x) &= \frac{d}{dx}[f'(x)] \\ &= \frac{d}{dx}(4 - \sec^2 x) \\ &= \frac{d}{dx}(4) - \frac{d}{dx}(\sec^2 x) \\ &= 0 - \frac{d}{dx}(\sec x)^2 \\ &= -2(\sec x)^1 \cdot \frac{d}{dx}(\sec x) \\ &= -2 \sec x \cdot (\sec x \tan x) \\ &= -2 \sec^2 x \tan x \end{aligned}$$

Part (b)

Below is a plot of the function and its first and second derivatives versus x.



Notice that $f'(x)$ is zero whenever the tangent line to $f(x)$ is horizontal, $f'(x)$ is positive whenever $f(x)$ is increasing, and $f'(x)$ is negative whenever $f(x)$ is decreasing. Also, $f''(x)$ is positive whenever $f(x)$ is concave up, $f''(x)$ is negative whenever $f(x)$ is concave down, and $f''(x) = 0$ whenever there's an inflection point in $f(x)$.